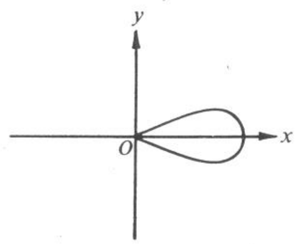
AP Calculus BC
Chapter 10 Part 2 – AP Exam Problems

All problems are NON CALCULATOR unless otherwise indicated.

1. The area of the region enclosed by the polar curve $r = 2\sin(2\theta)$ for $0 \le \theta \le \frac{\pi}{2}$ is

A) 0 B) $\frac{1}{2}$ C) 1 D) $\frac{\pi}{2}$ E) $\frac{\pi}{4}$



2. Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r = 4\cos(3\theta)$ shown in the figure above?

A) $16\int_{-\pi/3}^{\pi/3} \cos(3\theta)\, d\theta$

B) $8\int_{-\pi/6}^{\pi/6} \cos(3\theta)\, d\theta$

C) $8\int_{-\pi/3}^{\pi/3} \cos^2(3\theta)\, d\theta$

D) $16\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\, d\theta$

E) $8\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\, d\theta$

3. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ B) $3\int_0^{\pi} \cos^2\theta\, d\theta$ C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$

D) $3\int_0^{\frac{\pi}{2}} \cos\theta\, d\theta$ E) $3\int_0^{\pi} \cos\theta\, d\theta$

4. The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve $r = 2$ is given by

A) $\frac{1}{2}\int_0^{\pi} (4\sin\theta - 2)^2\, d\theta$

B) $\frac{1}{2}\int_{\pi/4}^{3\pi/4} (4\sin\theta - 2)^2\, d\theta$

C) $\frac{1}{2}\int_{\pi/6}^{5\pi/6} (4\sin\theta - 2)^2\, d\theta$

D) $\frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(16\sin^2\theta - 4\right) d\theta$

E) $\frac{1}{2}\int_0^{\pi} \left(16\sin^2\theta - 4\right) d\theta$

5. Which of the following expressions gives the total area enclosed by the polar curve $r=\sin^2\theta$ shown in the figure to the right?

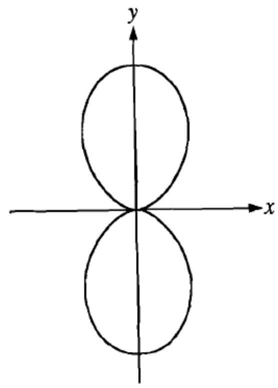
(A) $\frac{1}{2}\int_0^{\pi}\sin^2\theta d\theta$

(B) $\int_0^{\pi}\sin^2\theta d\theta$

(C) $\frac{1}{2}\int_0^{\pi}\sin^4\theta d\theta$

(D) $\int_0^{\pi}\sin^4\theta d\theta$

(E) $2\int_0^{\pi}\sin^4\theta d\theta$



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6. (1984 BC5) Consider the curves $r=3\cos\theta$ and $r=1+\cos\theta$.

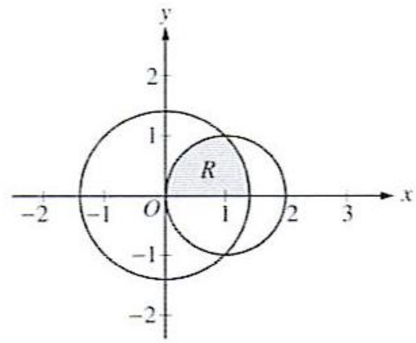
(a) Sketch the curves on a set of x and y-axes.
(b) Find the area of the region inside the curve $r=3\cos\theta$ and outside the curve $r=1+\cos\theta$ by setting up and evaluating a definite integral. Your work must include an antiderivative.

7. (1990 BC4) Let R be the region inside the graph of the polar curve $r=2$ and outside the graph of the polar curve $r=2(1-\sin\theta)$.

(a) Sketch the two polar curves on a set of x and y axes and shade the region R.
(b) Find the area of R.

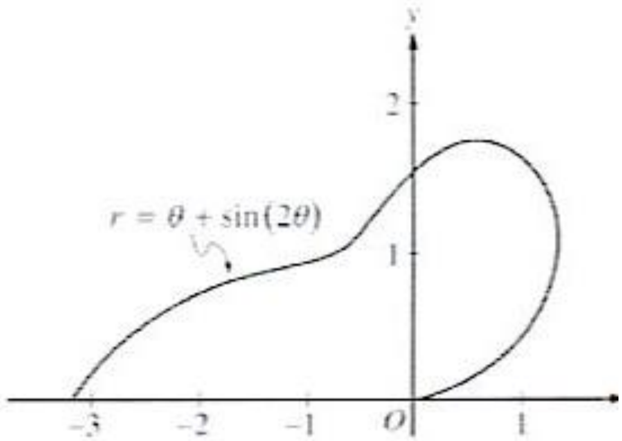
8. (1993 BC4) Consider the polar curve $r=2\sin(3\theta)$ for $0\le\theta\le\pi$.

(a) Sketch the curve on a set of x and y-axes.
(b) Find the area of the region inside the curve.
(c) Find the slope of the curve at the point where $\theta=\frac{\pi}{4}$.



9. (2003B BC2) The figure shows the graphs of the circles $x^2 + y^2 = 2$ and $(x-1)^2 + y^2 = 1$. The graphs intersect at the points (1, 1) and (1, −1). Let *R* be the shaded region in the first quadrant bounded by the two circles and the *x* – axis.

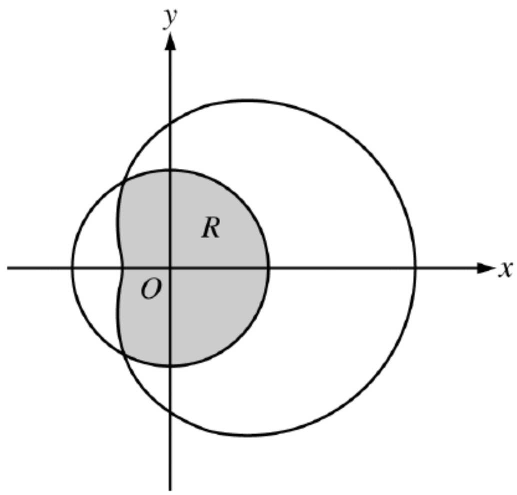
   (a) Set up an expression involving one or more integrals with respect to *x* that represents the area of *R*.
   (b) Set up an expression involving one or more integrals with respect to *y* that represents the area of *R*.
   (c) The polar equations of the circles are $r = \sqrt{2}$ and $r = 2\cos\theta$, respectively. Set up an expression involving one or more integrals with respect to the polar angle θ that represents the area of *R*.



10. (2005 BC2) The curve above is drawn in the *xy* – plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where *r* is measured in meters and θ is measured in radians. The derivative of *r* with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

    (a) Find the area bounded by the curve and the *x* – axis.
    (b) Find the angle θ that corresponds to the point on the curve with *x* – coordinate −2.
    (c) For $\frac{\pi}{3} \le \theta \le \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about *r*? What does this fact say about the curve?
    (d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

11. (2007 BC3) The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure below. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

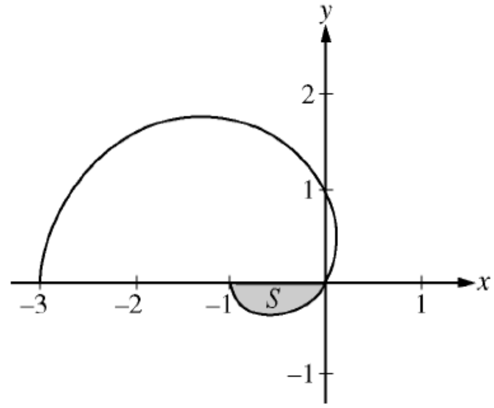


(a) Let R be the region that is inside both graphs. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. The particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

12. (2009B BC4) The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown below. Let S be the shaded region in the third quadrant bounded by the curve and the x – axis.



(a) Write an integral expression for the area of *S*.

(b) Write expression for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

(c) Write an equation in terms of *x* and *y* for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.

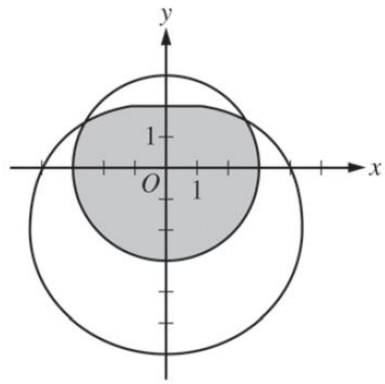
13. (2011B BC2) The polar curve *r* is given by $r(\theta) = 3\theta + \sin\theta$, where $0 \le \theta \le 2\pi$.

(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of *r*.

(b) For $\frac{\pi}{2} \le \theta \le \pi$, there is one point *P* on the polar curve *r* with *x*-coordinate −3. Find the angle θ that corresponds to point *P*. Find the *y*-coordinate of point *P*. Show the work that leads to your answers.

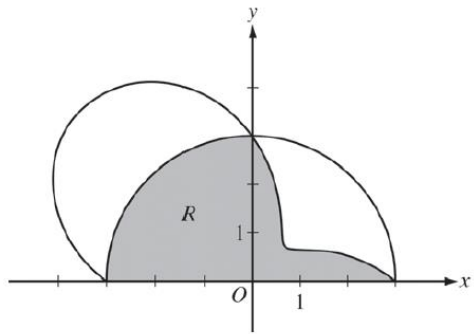
(c) A particle is traveling along the polar curve *r* so that its position at time *t* is $(x(t), y(t))$ and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

14. (2013 BC2) The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure below. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.



(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.
(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.
(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity at time t = 1.5.

15. (2014 BC2) The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure below for $0 \le \theta \le \pi$.

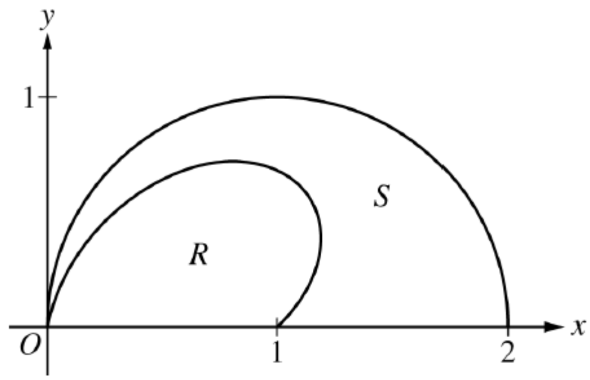


(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\dfrac{dx}{d\theta}$ at $\theta = \dfrac{\pi}{6}$.

(c) The distance between the two curves changes for $0 < \theta < \dfrac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \dfrac{\pi}{3}$.

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\dfrac{d\theta}{dt} = 3$ for all times $t \ge 0$. Find the value of $\dfrac{dr}{dt}$ at $\theta = \dfrac{\pi}{6}$.



16. (2017 BC2) The figure shows the polar curves $r = f(\theta) = 1 + \sin\theta\cos(2\theta)$ and $r = g(\theta) = 2\cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. Let *R* be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the *x* – axis. Let *S* be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the *x* – axis.

(a) Find the area of *R*.

(b) The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides *S* into two regions of equal area. Write, but do not solve, and equation involving one or more integrals whose solution gives the value of *k*.

(c) For each θ, $0 \le \theta \le \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A, the average value of $w(\theta)$ over the interval $0 \le \theta \le \frac{\pi}{2}$.

(d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ? Give a reason for your answer.

Answers

1. D	1985	BC	#24	41%
2. E	1988	BC	#23	55%
3. A	1997	BC	#21	22%
4. D	1998	BC	#19	37%
5. D	2008	BC	#26	38%